A control theoretical approach to crowd management: Comment on "Human behaviours in evacuation crowd dynamics: From modelling to "big data" toward crisis management" by Nicola Bellomo et al.

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The formulation of mathematical models for crowd dynamics is one current challenge in many fields of applied sciences. It involves the modelisation of the complex behavior of a large number of individuals. In particular, the difficulty lays in describing emerging collective behaviors by means of a relatively small number of local interaction rules between individuals in a crowd. Clearly, the individual's *free will* involved in decision making processes and in the management of the social interactions cannot be described by a finite number of deterministic rules. On the other hand, in large crowds, this individual indeterminacy can be considered as a local fluctuation averaged to zero by the size of the crowd. While at the microscopic scale, using a system of coupled ODEs, the free will should be included in the mathematical description (e.g. with a stochastic term), the mesoscopic and macroscopic scales, modeled by PDEs, represent a powerful modeling tool that allows to neglect this feature and provide a reliable description. In this sense, the work by Bellomo, Clarke, Gibelli, Townsend, and Vreugdenhil [2] represents a mathematical-epistemological contribution towards the design of a reliable model of human behavior.

Besides the exhaustive review work and the cognitive contribution to the modeling problem, the essay [2] also inspires new issues concerning the application to crowd management; for instance, the identification and the analysis of control mechanisms in crowd behaviors. Control problems in crowd dynamics represent a rather new topic, which has already attracted the attention of several scientists.

Following the characterization of the modeling scales given by [2], we distinguish between micro, meso, and macro models. On the other hand, concerning

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the control strategies, we identify three different mechanisms: the control via leadership, the ℓ_1-optimal sparse control, and the distributed action. At the microscale, the controllability of crowd models using distributed controls is a classical topic. However, the rational action of a controller is in contrast with the concept of crisis. It is therefore more interesting to study a control acting only on a subgroup of agents, or leaders, which mey represent trained crisis managers implementing the most appropriate action during an evacuation process. Recently, the leadership based controllability and optimal control of multi-agent systems has been proposed in [3] for second order models, in [12] for opinion formation models, and in [13] for evolving social networks. In [4] and [5] the authors proposed an efficient control mechanism acting only on the smallest number of agents, introducing sparse controllability problems. In [8] ideas from the kinetic theory of gases are used to propose mathematical models at meso and macro scales of multi agents opinion formation systems including a subgroup of leaders. Similar problems have been treated in [1]. At the macro scale the control by leadership has been investigated in [6]. The sparse controllability of a kinetic alignement model is proposed in [9].

The literature on the subject is growing rapidly and the present note presents a very brief and non-exhaustive discussion of results in this direction. However, we complete this discussion with a mention on possible future developments in mathematical modelling of crowd management problems, including, for instance, the role of the geometry of the domain (or more generally, the environment) in an evacuation process. On the one hand, the shape of the domain may amplify the stress conditions [11, 10], leading to the study of crowd-environment interactions, and on the other hand, shape-topology optimization problems can be formulated to design optimal spatial domains that facilitate the pedestrians in the adoption of a more rational behavior [7].

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