Keep right or left?
Towards a cognitive-mathematical model for pedestrians[*]

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Abstract

In this paper we discuss the necessity of insight in the cognitive processes involved in environment navigation into mathematical models for pedestrian motion. We first provide a review of psychological literature on the cognitive processes involved in walking and on the quantitative one coming from applied mathematics, physics, and engineering. Then, we present a critical analysis of the experimental setting for model testing and we show experimental results given by observation. Finally we propose a cognitive model making use of psychological insight as well as optimization models from robotics.

1   Introduction

To design safe, comfortable, and efficient environments for people, it is essential to understand how they move through space. As a consequence, designers and engineers have long been interested in pedestrian behavior. More recently, mathematicians, physicists, psychologists, and sociologists have turned their attention to this problem. It is not surprising then that the literature on pedestrian behavior spans a vast range of approaches, not only in the experimental methods and analytical tools that are used, but also in the types of questions that are asked.

Reviewing this vast literature is beyond the scope of this paper. Here we report some approaches focusing on pedestrians as individuals. Then it is of paramount importance to take into account the cognitive process involved in walking. In particular, this provides new insight with respect to mathematical modeling with focus on moving humans as "particle" or self-propelled agents. Such psychological components show up both in the choice of walking strategies and preferences and in interaction rules with other pedestrians.

The focus will be mainly on investigations addressing the natural behavior of the single pedestrian moving in an simple environment. The interaction rules between moving agents will not be addressed at the same level of details. We chose to study the natural behavior because pedestrians who are knowingly participating in a study may not behave as the normally would. As described later, our results bear out this concern. Moreover the dynamics of a single pedestrian in a simple environment are more easily described by mathematical models designed following mechanical constraints and optimization objectives. Our results will show, however, that these mathematical models do not capture the observed behaviors. Instead, we find that social and cultural factors influence pedestrian behavior even when the pedestrian is alone, unaware to be observed, and in a natural

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or familiar environment. Indeed mathematical models would be expected to accurately capture pedestrian behavior when the following assumptions hold: the pedestrians goal is to traverse the space as quickly and efficiently as possible, the pedestrians perception of the environment is accurate, and the pedestrian has no preferences for a particular side or direction. Each of these assumptions is violated in certain situations. For example, pedestrians in shopping malls, museums, and gardens are not always focused on traversing the space as quickly as possible. And pedestrians navigating a city may have highly incomplete or inaccurate representations of their environment. Ultimately, a full model of pedestrian behavior would need to address these diverse situations, but this is far beyond the current state of the field. Here we limit ourselves to the behavior of a single pedestrian in two very simple environments (a sidewalk and a stairwell), and for these situations the first two assumptions seem reasonable. We are interested in testing the third assumption, that is, whether pedestrians have a preference for a particular side or direction.

In the United States, the notion that people have a preference for the right side seems widespread, and so it is surprising that it is supported by relatively little empirical evidence.

Two explanations have been offered for the right side preference. One explanation focuses on the need for a social convention that allows on-coming pedestrians to act in concert to avoid collisions (see, for instance, [16, 30, 35]). According to this explanation, the side that the convention favors is arbitrary, and so one would expect some cultures to have a right-side bias while others have a left-side bias. The bias would spread through the culture primarily through social encounters, but it could also be promulgated through building conventions such as the direction of revolving doors and the arrangement of escalators and moving sidewalks. Side preferences do vary across countries, but overall there seems to be a stronger preference for the right (see the survey [21]).

The stronger preference for the right could reflect cross-cultural influences or it could be a consequence of brain lateralization (see, for instance, [34, 21]). Because of the functional asymmetries of the right and left cerebral cortices, most people show a number of right-side biases including right-handedness. Lateralization could produce a preference for the right side directly, or it could operate indirectly. For example, people who are right handed will typically carry objects (suitcases, briefcases, shopping bags, etc.) on the right side. To avoid bumping on-coming pedestrians with these objects they will favor passing on the right side.

If a right side preference is grounded in the avoidance of an on-coming pedestrian, it is unclear whether it would persist when a pedestrian is walking down an empty sidewalk or stairwell. With no social constraints on behavior, pedestrians may simply optimize their route for speed and ease. Alternatively, pedestrians may maintain a right side preference because of habit or because of the possibility encountering another pedestrian further down the path or around the corner.

To determine whether isolated pedestrians show a right side preference, we observed the behavior of 8540 pedestrians in two settings. The first setting was near the intersection of two broad sidewalks. Across many weeks we varied the position of a traffic cone on the sidewalk and observed whether pedestrians passed this cone on the right or left. The second setting was a staircase with an intermediate landing and a 90-degree turn. Each stair flight was bisected by a handrail, while the intermediate landing was open. We observed the side preferences of pedestrians ascending and descending the stairs.

We collected these data from a certain distance or with surveillance cameras, so the pedestrians were unaware of being observed. We were interested in whether we would obtain similar results if we had instead conducted a survey or an experiment. So, we carried out two additional studies in which we asked pedestrians either to predict their behavior or to allow us to observe their behavior. By comparing the results across studies, we could compare the validity of three common methods used in pedestrian research.

In view of the results of these experiment we finally propose a simple model combining the quantitative and qualitative aspects: an optimal control model used in robotics, see [1], with a cost functional depending on psychological bias as, for example, the tendency to keep the right side.

The structure of the paper is as follows. In Section 2 we discuss some models proposed by works in different fields, not limited to psychology. However, the latter are more of qualitative nature, as opposed to mathemati-

cally advanced ones. In Section 3 we will deal with experiments and measurements. In particular, in the first "pathway experiment" (see Section 3.2), we will discuss how the experimental setting influences results because of expected *psychological bias*. We will also compare different experimental settings, showing how sensitive to them measurements can be. In the second experiment, the "stairway experiment" (see Section 3.3), we will address the question of the criteria, both mechanical and psychological, affecting the pedestrians' choice when ascending or descending stairs. Finally, in Section 4, we will present a simple cognitive–mathematical model based on the cryptical analysis of the literature and on the observation of the experiments in the preceding sections.

2   Literature review

The study of pedestrian behavior has attracted the attention of different research fields, among which Applied Mathematics, Architecture, Biology, Cognitive Science, Medicine and physical therapy, Physics (such as particle systems and statistical mechanics), Psychology, Sociology, and Transportation engineering. Therefore, there is a widespread literature on the argument, unfortunately oftentimes with small or no intersection for what concerns the comparison of results or even just cross-referencing. For this reason, a complete literature review is not only outside the main scope of the present paper but also very challenging.

However, narrowing down the general scope of a literature review, we provide a brief description of the most influential mathematical models proposed in the literature (by engineers, mathematicians and physicists) and some features, studied in literature of different type, which are related to social and geographical issues and thus usually not included in mathematical models.

In this respect, at least a first classification of the existing research can be done by distinguishing *mathematical models* (e.g. by engineers, physicists, biologists, some of the psychologists, etc.) and *behavioral models* (e.g. by sociologists, architects and some of the psychologists). There exists a quite extensive number of papers in the mathematical literature, such as [8, 14, 31, 29, 28]. Among the more notable contributions on behavioral models of particular interest for our investigation, we cite [10, 12, 33, 9].

The common ground among these different contribution is represented by the interest in the characteristics of the movement of a single pedestrian (rather than a large crowd) and his/her interactions with the environment and the other pedestrians. As a side note, let us mention that also the early literature in transportation engineering oftentimes aimed at taking account social features in the study of pedestrian flows, such as purposes of walking, age, gender, size and others.

2.1   Quantitative mathematical models

Here we briefly describe four different mathematical models proposed in literature, everyone representing a different approach to the modeling problem. We refer to [7] for a complete review on pedestrian modeling and to [2] for traffic and crowds modeling.

In a number of models of crowd dynamics, the motion of the single pedestrian is reduced to the problem of determining a *preferred velocity* and then adding interaction terms with the other pedestrians or with obstacles. However, even if apparently a simple approach, this *ansatz* gave rise to a number of interesting models with the preferred velocity determined by solving optimization problems associated with various cost functions.

The social force model of Helbing and Molnar. In [15] Helbing and Molnar introduce a model in which the motion of a single pedestrian is related to a *social force*, representing the effect of the environment on the behavior of the pedestrian. The term "social" is to highlight the fact that the force is not excerted on the pedestrian's body but it rather describes the motivation to change the velocity of the pedestrian given by the perceived information about the environment. This concept of social force has been firstly introduced in [13].

The dynamics of a pedestrian α is given by a nonlinearly coupled Langevin equation of the form

$$
\begin{cases}
\dfrac{d}{dt}\vec{r}_\alpha(t) = \vec{w}_\alpha(t)g_\alpha(t), \\
\dfrac{d}{dt}\vec{w}_\alpha(t) = \vec{F}_\alpha(t).
\end{cases}
$$

Where the position and velocity of the pedestrian α at time t are described by the vectorial quantities $\vec{r}_\alpha(t)$ and $\vec{w}_\alpha(t)$ respectively. The scalar function g_α is a cut-off function bounding the velocity of the pedestrian with the maximal velocity $v_\alpha^{\max}$, namely

$$
g_\alpha(t) = \begin{cases}
1 & \text{if } \|\vec{w}_\alpha(t)\| \leq v_\alpha^{\max}, \\
v_\alpha^{\max}/\|\vec{w}_\alpha(t)\| & \text{otherwise.}
\end{cases}
$$

The vectorial quantity $\vec{F}_\alpha(t)$ is the social force governing the evolution of the pedestrian's velocity. This force describes three types of different effects on the pedestrian's behavior: the *goal*, the attractive effects, and the repulsive ones. The first term depends on the position $\vec{r}_\alpha$ and on the velocity $\vec{w}_\alpha$ and it accounts for the will of the pedestrian to reach a desired position with a desired velocity, possibly taking the shortest way. The repulsive terms account for the repulsive effects given by the presence of other pedestrians and borders (walls, obstacles). Similarly the fact that the pedestrian is attracted by other persons or objects (windows, landmarks) is modeled by the attractive terms. Both the repulsive and the attractive forces depend on the position $\vec{r}_\alpha$ and may contain a direction dependent factor accounting for the vision cone of the pedestrian.

Optimal control model. An influential mathematical model describing the human locomotion has been showed in [1]. The authors suggested that the human locomotion is governed by a differential controlled system with an optimal constraint. This observation is based on an experiment of motion tracking for single pedestrians free to move in a big environment, a large gymnasium. The locomotors were asked to naturally walk from a fixed starting position to a porch without any spatial constraints relative to the path. The porch has been placed in 40 different positions and 12 directions for each position on a grid covering the gymnasium. The object was to cover the 3 dimensional space of positions and directions. More than 1500 trajectories has been recorded using 34 light reflective markers located on the body of each of 7 locomotors. The state of the locomotor is described by four variables. The middle point of the torso, between left and right shoulder is represented by the 2-dimensional coordinates on the plane (x_T, y_T), the direction of the torso by φ_T, and the curvature of the torso by κ_T. The dynamics obeys to the nonholonomic system given by

$$
\begin{pmatrix} \dot{x}_T \\ \dot{y}_T \\ \dot{\varphi}_T \\ \dot{\kappa}_T \end{pmatrix} = \begin{pmatrix} \cos\varphi_T \\ \sin\varphi_T \\ \kappa_T \\ 0 \end{pmatrix} u_1 + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u_2. \tag{1}
$$

where the scalar functions of time $u_1 : [0,\infty) \in [a,b]$ and $u_2 : [0,\infty) \in [-c,c]$ are the control parameters, accounting for the linear velocity and the time derivative of the curvature, respectively. The motion minimizes a linear combination of this two quantities,

$$
\min \quad \frac{1}{2}\int_0^{t_f} \left(\alpha u_1^2 + \beta u_2^2\right) dt, \tag{2}
$$

with the two real parameters α and β depending on the physical characteristics of the pedestrian.

The solution of the optimal control problem (1)-(2) is a concatenation of arcs of clothoid (or cornu spiral) and provides a very good approximation of human walking paths. Using similar inverse optimal control techniques, the model has been refined allowing lateral movement in later works [25, 24].

Hoogendoorn and Bovy's model. The Hoogendoorn and Bovy's microscopic model [17] is based on the assumption that pedestrians can forecast to a certain extent the behavior of the others, and then choose their direction of motion on the basis of the forecast. It consists of two main ingredients: A force-based model and a cost functional to be minimized, which translates the "cost" (in terms of discomfort due to proximity of other pedestrians, straying from the desired direction, etc.) associated to every possible trajectory joining the current position to the desired target. Considering a system of N pedestrian the model for each one has the form

$$\begin{cases} \dot{X}_t^k &= V_t^k \\ \dot{V}_t^k &= F_t^k + U_t^k \end{cases}$$

for $k = 1, \ldots, N$, where U_t^k is a control variable which can be freely chosen by each pedestrians in a given set of admissible controls $\mathcal{U}_k$.

Mean field game models. The mean field game approach assumes pedestrians to be rational and have rational expectations [22]. Individuals anticipate the crowd evolution first, then evaluate their cost function. Next, they deduce their strategy (feedback control). Finally, the mass evolves according to these strategies. At the optimum the mass evolution has to coincide with the one which has been anticipated, according to the rational expectations assumption. The crowd strategy is then a Nash equilibrium. Considering a continuum of pedestrians, or players, we present the equations of the mean field game system whose solutions are mean-field equilibria. The two-dimensional first order stochastic dynamics of a single pedestrian located at some generic point $x \in \mathbb{R}^2$ is

$$\begin{cases} dX_t &= U_t dt + \sigma dW_t, \quad t \in [0, T] \\ X_0 &= x, \end{cases}$$

where T is the final time, W_t is the two-dimensional Brownian motion, $U_t \in \mathcal{U}$ is the control variable, and $\mathcal{U}$ is the set of admissible controls.

2.2 Qualitative models

The models presented in the previous section are quantitative or, more precisely, are mathematical models and provide equations which should predict the behavior of crowd motions. On the other side a number of researchers studied the pedestrian behavior deducing some general behavioral rules. Out of the wide literature on the more behavioral side, we will focus on three main subjects:

a) Cognitive maps.

b) The choice of a preferred path.

c) Geographical features.

Cognitive maps. It is know that human walk is a complicate task involving a number of cognitive processes in it. In particular, the navigation of environments is operated making use of virtual maps, created in our brain, which are representations of the reality. Such maps are called *cognitive maps*.

Once cognitive maps represented the only tool at disposal of the walking human, while nowadays a number of technological devices (such as GPS) provide additional guidance. Nevertheless, it is still important to understand the subconscious process behind the creation of cognitive maps, for instance to understand possible distortions

and defects. A cognitive map is oftentimes created starting from direct observation (or measurements) of the real environment or could be created off-line by using traditional maps. We can thus distinguish two main modes of information acquisition: *route-based knowledge* and *survey knowledge*. In any case, the process is dynamic and information is continuously acquired to adjust the map as the navigation proceeds. Moreover, the process uses different sources and may be not uniform in time. The walking pedestrian uses sensorimot apprehension to infer information about the ambient space. The corresponding learning process was subject of various studies, which indicated significant differences among subjects.

The resulting cognitive maps can present a number of errors and distortions with respect to the reality. In particular the following effects are reported in various studies: distances are usually overestimated with less distortion for shorter distances, rotations occur to align with landmarks, home location affects the whole process. For a more extensive discussion we refer the reader to [23, 11, 26, 19].

It is interesting to notice that technology, remarkably GPS, seems to slow down or anyhow negatively effect the learning process. The final result included slower motions and longer distances with respect to those achieved using a direct knowledge. See [18].

Path choice. The final result of the use of a cognitive map is the selection of the "best" path to use. The selection process was mostly studied in condition of no interaction with other pedestrians and the following selection criteria were observed:

Optimization criteria shortest or minimum time path, shortest or longest leg first, straighter leg first.

Sensational criteria most scenic, aesthetically more valuable, less obstacles.

Comfort criteria fewest turns, minimal angular deviation.

Knowledge criteria most known, first noticed, different from previously taken.

The results of experiments were oftentimes not definitive, with criteria which may contrast each other. Therefore these criteria should be considered with care when designing a model. For simple environment geometry many may concur to the selection of the same path, thus in those cases are of value for the modeling activity.

The criteria listed above represent just a sample of those considered in the literature. Other examples are visibility, chance of accidents, patrol by authorities, safety in general and other. Moreover, the different criteria can be correlated: lack of visibility may impair the detection of information for the path selection thus affecting the optimization criteria.

One has also to notice that most experiments were performed by interview showing a map (real or virtual) to subjects. The limitations of such approach are discussed in Section 3.

Some quantitative results are also available, with measurement induced by choices taken at critical points of the path, such as crossing and junctions. However, also in this case the results were rarely conclusive, but rather indicated the difficulty of describing the complicate process of path selection by means of few simple rules. For more extensive discussion of path choice see [11, 6, 32].

Geographical and social features. In many experiments, it was observed the high level of efficiency of pedestrian flow in structured environment. Thus there is a quite general agreement on the fact that cooperation represents one of the main feature of walking in interaction with other pedestrians. One example, experience by most people every day, is the clear organization in lanes, both for the case of unidirectional flows or opposite flows. In this respect, the choice of the walking side is critical for the good functioning of the overall flow. Moreover, such choice can be considered as one example of *social rule* and depends on the geographical location.

It was notice that in central Europe pedestrians exhibit a preference to walk on the right-hand side. In this case there is a correspondence with the driving rules. Such correspondence is violated in Great Britain, where people prefer the right-hand side when walking, opposed to the left-hand side rule for roads.

Such picture is completed by realization of the other two possibilities. In Japan both cars and pedestrians stay on the left-hand side, while in Korea the opposite of Great Britain happens: cars drive on right-hand side and pedestrian usually choose left-hand side.

The lane formation is a quite robust phenomenon observed ubiquitously in most countries. However, a precise quantification of the phenomenon is still lacking and is mostly noticed when pedestrians are interacting, i.e. for large enough crowds.

The emergence of lane formation or other organized structure is often referred to as *self-organization*. More generally, self-organization is the emergence of patterns in large groups by means of simple interaction rules, with oftentimes the single individual not able to observe the overall group formation. One may question if self-organization and efficiency are the cause or consequences of the social rules, but a definite answer was not given. However, interesting discussion included the consideration of differential games models.

For a complete account of preferences in walking sides and their geographical distribution we refer to [8, 14] and references therein.

3   Experiments

In this section we briefly describe the different experimental settings commonly used in the literature and their influence on the pedestrians' behavior, then we present in details two experiments: the first one, in Section 3.2 to analyze the influence the knowledge of being in an experiment has on the behavior of a pedestrian; the second one, in Section 3.3 to show how mathematical models should take into account both of the mechanicistic and the cognitive aspects of the human locomotion.

3.1   Experimental settings

The cognitive aspects underlying walking in humans, see Section 2.2, render of paramount importance an accurate choice and preparation of the experimental setting as well as the careful interpretation of the obtained data. For both aspects psychological biases have to be taken into consideration. Similarly the composition of pedestrian crowds involved in the experiments, in terms of age, gender, and trip purpose, must be chosen to be compatible with the experimental purposes and the local population composition. A large majority of experiments found in literature suffer of various limitation in the validity of their results due to such difficulties and some researcher even excluded the possibility that laboratory settings can guarantee any relevance of the experiments. Here we focus on the analysis of four experimental settings: interviews, virtual reality, artificial environments, and natural environments. The importance of introducing cognitive aspects in mathematical modeling was also pointed out recently in [27].

Interviews. Knowledge on some aspects of the behavior of pedestrian can be obtained by interviews. Generally speaking individuals will be exposed to a real environment (or a map) and will be asked about choices they would perform in that environment. It is well agreed that the process of walking happens at unconscious level, and so there are serious limitation of this tool. Indeed the pedestrian's behavior is strongly biased by the knowledge of being in an experiment.

Virtual reality. The studies performed by psychologists and cognitive scientists were oftentimes based on virtual reality. The main reason for this is the possibility to test many different landscapes and environments at the same time and with a relatively small cost. Moreover, the versatility of this environment allows to test many different hypothesis with a considerably simple modification of the experiments. However psychological bias is a serious risk and researchers usually need to somehow filter the information.

Artificial environments. Large experiments, involving a high number of pedestrians, may not be feasible or even dangerous in real environments. Moreover, the use of advanced tech instruments may be impossible. In particular, it is hard to find suitable places allowing one to observe both dense and undisturbed pedestrian

flows, and sensors (or cameras etc..) may not be easily placed. These are the main reason why many researchers resort to artificial environments: usually lab space equipped with cameras and with features resembling real environments.

The counterpart is an high risk of psychological bias, thus various ways to reduce it were considered: Not communicating the experiment purpose, giving some time to participants in the experiment to familiarize with the setup, providing routes and goals so to mimic a real life situation. Nevertheless in this case major psychological biases are unavoidable. For reference about experiments in artificial environment see in particular [14, 8].

Natural environments. Natural environments can be considered for experiments by using surveillance cameras (opposed to cameras placed in lab experiments as for artificial environments). The main advantage of such approach is the direct observation of pedestrians behavior, thus overcoming most of possible psychological bias. In particular this allows the testing of specific hypothesis. However, the application of devices on participants' bodies to perform precise measurement, as for artificial environment, may spoil the advantage of being in a natural environment. For the increasing number of available surveillance cameras, the improvement of their resolution and overall recording quality, and the fast development of image analysis softwares the use of cameras in natural environment will play a key role for future experiments. Currently, there are few known studies using surveillance cameras, we refer, for instance to [3] and [20].

Summing up, researchers are aware of the differences arising naturally between artificial or virtual environments on one side and the natural one on the other side, and limiting seriously the reliability of precise measures (leaving still some validity for the observation of some aggregate phenomena). For this reason the two experiments presented in the this section have been set-up in a natural environment. We report some results of direct observations and of surveillance cameras from a project performed on the Camden Campus of Rutgers - The State University of New Jersey. Thousands of students walk on the Campus area of Rutgers - Camden every day. The area is equipped with surveillance cameras and the public is informed by signs. However, students, faculty and staff walk on the Campus area every day during the academic year, thus they are not expected to pay much attention to the cameras (which are placed in elevated spots) or to exhibit unusual behavior because of that.

3.2 The pathway experiment: On the influence of the experimental setting

In this section we present results of an experiment conducted on a natural environment, that is Camden Campus of Rutgers - The State University of New Jersey. The purpose of this study is to analyse the psycological bias affecting the pedestrians' behavior in a natural environment.

3.2.1 Method

Location. The chosen location was the path leading from Armitage Hall towards the Campus Center on the Rutgers University-Camden Campus. This location was chosen because there is a clear entrance and exit to the path and, in particular, pedestrians could not leave the path until after passing by the obstacle. This path is also clear of other obstacles so as not to be confounded with the control obstacle. Moreover, this location had enough regular pedestrian traffic throughout a typical day so that adequate data collection was possible. However, the path remained usually relatively un-crowded increasing the possibility to measure "clean" choices of pedestrians, that is, not biased by interactions with other pedestrians. Its width is 315 cm and there is a 135-degree curve leading into the path to Armitage Hall. The path is surrounded by grass with no bushes or trees in the surroundings of the turn.

To have simple measurements two large orange traffic cones (around 90 cm tall) were placed next to each other at various points along the selected path, thus pedestrians were forced to choose to pass to the left or the right of the obstacle.
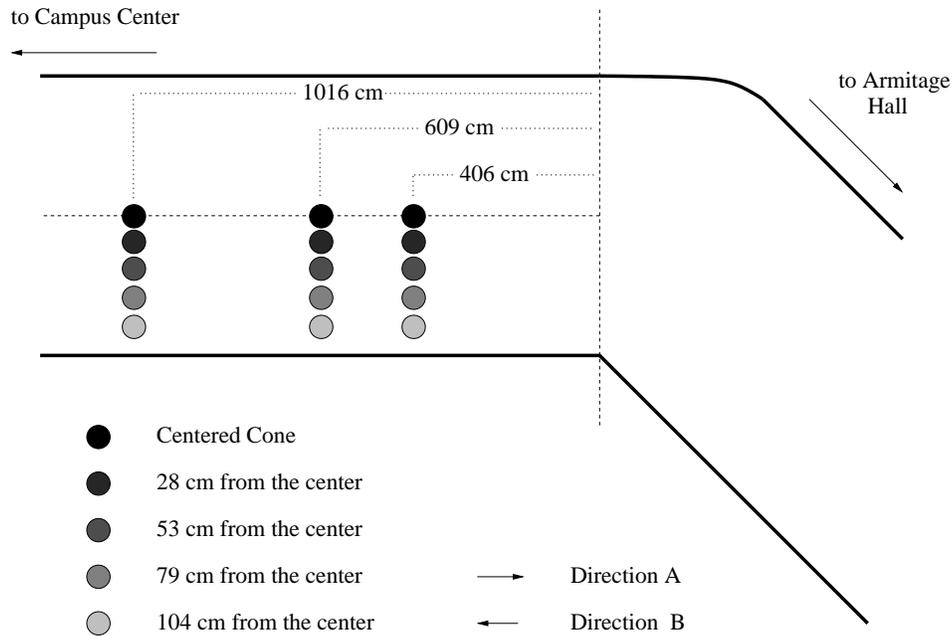
Figure 1: Position of the traffic cone on the path with respect to the beginning of the path (denoted by the vertical dashed line) and the center of the path (denoted by the horizontal dashed line).

In order to determine which way pedestrians naturally traveled the path, baseline data was collected with the cones directly in the center of the path. Then, various points along the path were determined in order to allow for a proper distribution across the path. The location of the cone placements were labeled in terms of their relative distance from the center of the path (or their location across the width of the path), as well as their distance from the corner of the path (or their location along the length of the path). A total of 15 cone positions were sampled altogether, including the initial baseline data. The distances across the path included 0 cm from the center (centered), 28cm from center, 53cm from the center, 79 cm from the center and 104 cm from the center. The distances along the path were 406 cm from the beginning of the path, 609 cm from the beginning of the path, and 1016 cm from the beginning of the path (see Figure 1).

For each of the 15 total cone positions, data from around 200 pedestrians was obtained. Data were collected for pedestrians walking in both directions. The direction in which pedestrians encounter the obstacle before the turn (thus from the Campus Center towards Armitage Hall) is coded as "Direction A", the other direction is coded as "Direction B".

Measures. Only data from pedestrians walking alone on the path, with no other pedestrians coming from either direction, and not carrying heavy objects were recorded. The experiment was performed in three settings:

Setting A Pedestrians were observed while walking on the path. They were not aware of being observed.

Setting B Pedestrians were informed that they were going to be observed meanwhile walking but not informed about which measurements were going to be taken.

Setting C Pedestrians walking on the path were stopped and interviewed about the choice they would have made with respect to passing the obstacle.

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Data have been collected during Summer 2011 for what concerns Setting A and during Fall 2011 for Setting B and Setting C. In order to avoid an influence of the season on the data we choose a path free of plants. Moreover we performed our experiments in relatively calm periods of the academic year.

3.2.2 Results

Table 1: Setting A

Direction A					
Distance from the center	Centered	28 cm	53 cm	79 cm	104cm
Distance from the beginning	406 cm				
Left	189	150	121	55	2
Right	11	50	79	145	198
Distance from the beginning	609 cm				
Left	168	159	117	34	4
Right	32	41	83	166	196
Distance from the beginning	1016 cm				
Left	170	142	123	36	3
Right	30	58	77	164	197

Table 2: Setting A

Direction B					
Distance from the center	Centered	28 cm	53 cm	79 cm	104 cm
Distance from the beginning	406 cm				
Left	54	89	148	188	198
Right	146	141	55	14	2
Distance from the beginning	609 cm				
Left	137	167	170	192	198
Right	85	44	32	9	2
Distance from the beginning	1016 cm				
Left	96	124	162	190	200
Right	155	76	38	10	0

The first set of data, concerning pedestrians not aware of being observed (Setting A), contains the right/left choice of 6122 pedestrians (3165 males and 2957 females) walking in both directions and with the cone in each of the 15 mentioned positions. The result are displayed in Table 1 for what concerns people walking in the Direction A and in Table 3.2.2 for Direction B. For the sake of readability we show the data with no distinction of gender, moreover we found this feature not statistically significant. In Table 3 there are the percentages of people passing to the right.

For what concerns Setting B (subject aware of being in an experiment although not informed about the measurements taken) a set of data of 100 pedestrians walking in the Direction B has been collected. The cone was in the center of the path and posed at 406 cm from the beginning of the path. The result of the observation is that 52 pedestrians walked on the right side of the cone while 48 on the left (see Table 4).

Table 3: Setting A. Percentage of people passing to the right

Direction A					
	Centered	28 cm	53 cm	79 cm	104 cm
406cm	95 %	75 %	61 %	28 %	1 %
609 cm	84 %	80 %	59 %	17 %	2 %
1016 cm	85 %	71 %	62 %	18 %	2 %

Direction B					
	Centered	28 cm	53 cm	79 cm	104 cm
406cm	27 %	39 %	73 %	93 %	99 %
609 cm	62 %	79 %	84 %	96 %	99 %
1016 cm	38 %	62 %	81 %	95 %	100 %

Table 4: Setting B

Direction	B
Distance from the beginning	406 cm
Distance from the center	Centered
Right	52
Left	48

Table 5: Setting C

Direction	A	B	B	B
Distance from the beginning	406 cm	406 cm	406 cm	1016 cm
Distance from the center	Centered	Centered	53 cm	Centered
Right	99	124	127	109
Left	1	76	30	46

The last set of data, taken on Fall 2011, concerns interviews about the choice the pedestrians would have made with respect to passing the obstacle (Setting C). A total of 612 subject have been interviewed (305 male and 307 females) and, in Table 5, data are grouped for each of the four selected positions of the obstacle.

3.2.3 Discussion

As already mentioned there is no statistically significant difference between the choice made by males and females, in none of the three settings.

In Setting A, it is remarkable that the choice of left and right is not symmetric and, in fact, it is highly a-symmetric. For the size of the path and the relatively wide turn one can expect a direct relation between the number of subjects passing to the right in the direction A and the number of subject passing to the left in the direction B. On the contrary this difference is, in general, statistically extremely significant. In Table 6 the p-value in the χ^2 for the comparison of number of subjects passing to the right of the cone in the direction A and to its left in the direction B. Except for the extremal case in which the cone was 104 cm far from the center of the path (last column of Table 6), leaving only slightly more than 50 cm for passing to the right for subjects walking in the direction A (to the left for subjects walking in the opposite direction) in any other case the difference is statistically significant. Even when the cone is very far from the corner (1016 cm) and pretty

close to the border (79 cm, that is a quarter of path width) the difference between the number of subjects to the right in the direction A and the number of subject passing to the left in the direction B is significant with a p-value in the χ^2 test of $4.6 \cdot 10^{-5}$.

Table 6: Setting A. p-values in the χ^2-test: Direction A vs Direction B

	Centered	28 cm	53 cm	79 cm	104 cm
406cm	$< 10^{-5}$	$2.4 * 10^{-3}$	$< 10^{-5}$	$< 10^{-5}$	1
609 cm	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$5 * 10^{-5}$	0.4
1016 cm	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$4.6 * 10^{-5}$	0.08

This asymmetry can be a result of several factors that can be either sociological (keeping the right) or mechanical (optimality of the trajectory, etc...). In Section 3.3, with another experiment in a natural environment, we address the problem of the identification of criteria involved in the right/left choice and of the selection of the weight to give to each of these factors.

Table 7: χ^2-test : Setting A vs Setting B

Position		Setting A (no.)	Setting B (no.)	Setting A (%)	Setting B (%)	p-value
Direction B	R	54	52	27	52	$1.9 * 10^{-5}$
406 cm, centered	L	146	48	73	48	

Comparing the data of the natural setting (Setting A) and the behavior while knowing to be in an experiment (Setting B) we have that the knowledge to be in an experiment changes radically the behavior of the pedestrian. Indeed, the difference between the results in setting A (for the cone in the same position) and Setting B is statistically extremely significant, with a p-value smaller than $2 \cdot 10^{-5}$ (see Table 7).

Table 8: χ^2-test : Setting A vs Setting C

Position		Setting A (no.)	Setting C (no.)	Setting A (%)	Setting C (%)	p-value
Direction A	R	189	99	94.5	99	0.06
406 cm, centered	L	11	1	5.5	1	
Direction B	R	54	124	27	62	< 0.0001
406 cm, centered	L	146	76	73	38	
Direction B	R	148	127	72.9	80.9	0.076
406 cm, 53 cm	L	55	30	17.1	19.1	
Direction B	R	96	109	38.2	70.3	< 0.0001
1016 cm, centered	L	155	46	61.8	29.7	
Total	R	487	459	57	75	< 0.0001
	L	367	153	43	25	

This is confirmed by the third setting, Setting C. As showed in Table 8 in general there is an extremely statistically significant difference between the results in the two settings. When interviewed, the majority of people stated that they were going to pass to the right of the obstacle, as the convention in United States suggests. However, if not observed, with the obstacle in central position, most of the people walking in the

Direction B cut the angle passing to the left of the obstacle. Table 8 shows the details of the χ^2-test comparing the results in Setting A and Setting C. In Direction A, since passing to the right of the obstacle was both conventional and shorter this difference is considered to be not quite statistically significant. The same is true in the case in which the obstacle was placed 53 cm away from the center.

3.3 The stairways experiment: On path choice

As already pointed out that the behavior of pedestrians is influenced by the knowledge to be part of an experiment and in Section 3.2, we show an example of this phenomenon. It follows the importance to conduct experiments in natural environments, without informing the people involved. In this section we study the behavior of pedestrians in a natural environment and we observe which are the factors involved in the left/right choice. In particular we propose two "optimality" criteria and we compare them with the "conventional" criterium of choosing the right side.

3.3.1 Method

Location. The experiment took place on the stairway in the middle of the Campus Center building, on the Rutgers University-Camden Campus. This staircase consists of two flights separated by a landing before a 90-degree turn, making an L-shape. Each flight is separated, in its whole length, into two sides by a handrail directly in the center committing the pedestrians to a choice (left or right). The presence of this handrail permitted to avoid introducing "unnatural" obstacles in the experiment. The experimental environment is depicted in Figure 2.

The starting location, the stair route, and the ultimate destination were recorded for each pedestrian. For pedestrians going downstairs, the starting location included either coming from the right (R), middle (M) or left (L), and ultimate destination could either be going to the right (R) or the left (L). This is reversed for pedestrians going upstairs. The side chosen on both flight (L or R), was recorded.

Measures. Only data from pedestrians traveling the staircase alone, with no other pedestrians traveling either up or down the stairs, were recorded. Both pedestrians going up the staircase and down the staircase were recorded. The data were obtained using direct observation, without the consent or knowledge of the pedestrians. The right(R) or left(L) labels are relative to an observer looking at the staircase as in the picture in Figure 2. Therefore the observer's perspective was aligned with the pedestrians walking upstairs and opposite with respect to the pedestrians walking downstairs.

3.3.2 Results

A total of 1706 pedestrians were recorded during two different sessions. A first set of measurements, taken in Spring 2011, gives data on the directions of 1058 subjects (472 going downstairs), however the direction upstairs has not been recorded. The second set of data, collected during Fall 2011, contains information on 648 pedestrians (328 going downstairs). Data are displayed in Table 3.3.2. Rows contain data grouped by starting position, direction, and ultimate destination of the subjects. For pedestrians going downstairs, the starting location included either coming from the right (R), middle (M) or left (L), and ultimate destination could either be going to the right (R) or the left (L). This is reversed for pedestrians going upstairs. In the table "down" means "going downstairs" while "up" means "going upstairs". For instance, "L down R" means coming from Left, going downstairs then taking the direction Right. Columns contain the options for stair route: Right-Right (RR), Right-Left (RL), Left-Right (LR), or Left-Left (LL) where the letter denotes the side of the first flight taken (the upper one for people going downstairs and the lower one for people going upstairs).
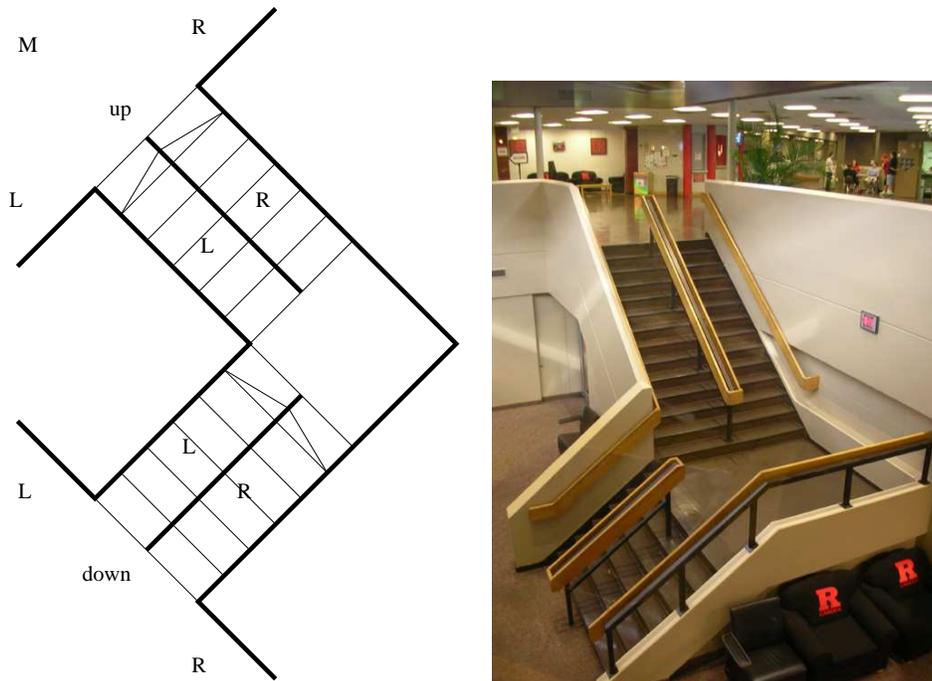
Figure 2: A schematic plan (on the left) of the stairways at Campus Center (on the right). The letters L, M, and R, respectively denoting Left, Middle or Right, may denote the direction upstairs (up), the direction downstairs (down), or the side of a flight.

3.3.3 Discussion

Greedy vs Conventional. As a first remark we observed that, in this setting, the conventional choice of taking the right flight (with respect to the subject) was very important. Indeed since every flight of stairs is divided, in its whole length, by the handrail people are forced to choose the right side to avoid possible pedestrians coming in the other direction. Another criterium observed is the "greedy" one, that is to choose the closest flight while entering the stairways. A significant number of people made this choice even when it is opposite to the conventional one. The two strategies are compared in Table 9 in which data are grouped independently on the direction of exit of the stairs.

The number of people applying a greedy strategy is comparable with the number of people making a conventional choice. Indeed the 46.5% of people going down and coming from right chose the right flight instead of the conventional left one. Similarly, the 58% of people coming from the left downstairs chose the left flight. Summing up these data we have that the 55% of people made a greedy-nonconventional choice. We can state that statistically the "greedy" factor is as important as the conventional.

Moreover these two factors are both relevant in the choice of the path. When these two factors coincide the choice is made by a large majority. Indeed 94% of subjects coming from left upstairs chose the conventional (and greedy) left flight and the 89% coming from right downstairs the conventional (and greedy) right flight.

Optimal vs Conventional. Another significant criterium observed is about the choice of the second flight when descending. When coming down the inertia is such that it is in some sense disadvantageous to make a

1st data set - Spring 2011				
	LL	LR	RL	RR
down R	125	33	22	107
down L	72	15	40	58
L up	68	37	21	66
R up	38	14	90	252
2nd data set - Fall 2011				
	LL	LR	RL	RR
L down R	9	7	0	1
L down L	30	1	1	1
M down R	36	18	1	19
M down L	69	2	8	11
R down R	8	11	1	32
R down L	41	1	15	5
L up L	6	6	1	9
L up M	57	10	2	42
L up R	18	15	0	15
R up L	2	0	1	21
R up M	4	0	6	57
R up R	1	1	2	44

Table 9: Greedy vs Conventional

	LL	LR	RL	RR
L down	39	8	1	2
R down	49	12	16	37
L up	149	68	24	132
R up	45	15	99	374

sudden turn, making a large turn on the landing a more preferable choice. This behavior can be the result of some optimization criteria (²/₃ power law, least angular momentum, etc.) which is hard to tell in such a natural setting. With an abuse of language we call this factor "optimal".

Table 10: Optimal vs Conventional

	LL	LR	RL	RR
Down L	212	19	64	75
Down R	178	69	24	159
Total	478		322	

While coming down, the optimal strategy for a pedestrian on the right side of the upper flight is to keep the right side on the second one independently on the direction to take downstairs. For a pedestrian on the left side of the upper flight it is optimal to switch to the right one to go in the direction right downstairs (in order to make a larger turn on the landing) and to keep the left side otherwise (in order to avoid a larger turn once downstairs). Table 10 shows the number of people going downstairs with no distinction on the starting locations.

The 64% of people used this optimization criterium to choose the side of the second flight, while the 60%

made the conventional choice (left side). Finally, it is significant that the 73% of pedestrians coming from the right side of the first flight took the "optimal" right flight independently on the arriving direction and only the 27% made a conventional choice.

4 A cognitive–mathematical model

In the sections above, and in particular with the results of the pathway experiment in Section 3.2, we show how the pedestrian behavior is strongly influenced by the knowledge to be in an experiment. This is an actual limit to the ability to represent accurately the pedestrian behavior using only mechanical/physical assumptions and to the validation of these models with the use of sensors or artificial environment. An accurate model should consider also the bias affecting the pedestrian behavior and its trajectories in a natural environment. In this section we briefly present a simple model including psychological/sociological effects.

Let us consider the optimal control problem (1)-(2) representing, as showed in [1], a good approximation of human walking path in an experimental setting. The motion of a pedestrian walking in a natural environment is governed by the same dynamical constraints as one moving in an experimental setting, while the psychological effect should enter in the cost given to certain actions or choices.

We consider therefore an adapted version of the Optimal control model (1)-(2) presented in Section 2.1

$$\min \quad \frac{1}{2} \int_0^{t_f} \left(\alpha u_1^2 + \beta u_2^2 \right) dt + \gamma \int_0^{t_f} \psi(X_0, X_f, X, t) dt, \tag{3}$$

$$\text{subject to} \quad \begin{aligned} \dot{x}_T &= u_1 \cos \varphi_T \\ \dot{y}_T &= u_1 \sin \varphi_T \\ \dot{\varphi}_T &= u_1 \kappa_T \\ \dot{\kappa}_T &= u_2, \end{aligned}$$

where $X = (x_T, y_T, \varphi_T, \kappa_T)$ and X_0 and X_f denote the initial and final conditions respectively. The cost functional ψ, accounting for the nature of the bias, depends *a priori* on the initial and final conditions. For example the cost functional

$$\psi(X_0, X_f, X, t) = (y_T(t_f) - y_T(0))(x_T(t) - x_T(0)) - (y_T(t) - y_T(0)(x_T(t_f) - x_T(0)),$$

penalizes the left side of a path forcing the trajectories to "keep the right".

In general there can be a number of qualitative criteria affecting the human locomotion and each of them can be represented by adding an associated cost weighted by a constant γ that can be tuned in function of the strength of their effects. Finding the good cost with the good parameters is a problem usually called "inverse optimal control" problem, that is: starting from a set of trajectories the problem find the optimal control problem whose solutions are the given trajectories. There are several recent studies in this field (see, for instance, to [4, 5]).

To show how in model (3) the addition of a term accounting for the tendency of "keeping the right" works we consider the following example: Consider admissible controls $u_1 \in [-1, 1]$, $u_2 \in [-2, 2]$ and parameters $\alpha = \beta = 2$. For a small $\varepsilon > 0$ consider the initial condition $X_0 = (0, 0, \pi/2 + \varepsilon, 0)$ and final condition $X_f = (1, 0, \pi/2 - \varepsilon, 0)$. The constraint on the state: $(x_T - 0.5)^2 + y_T^2 \geq 10^{-8}$ models an obstacle (say a traffic cone). In this setting the pedestrian is forced to make a choice between passing to the left of the obstacle or to the right. Of course, with these initial and final condition the optimal "purely mathematical" path (i.e. with $\gamma = 0$) is given by the solide line (Figure 3): the pedestrian passes to the left of the obstacle. If we add to
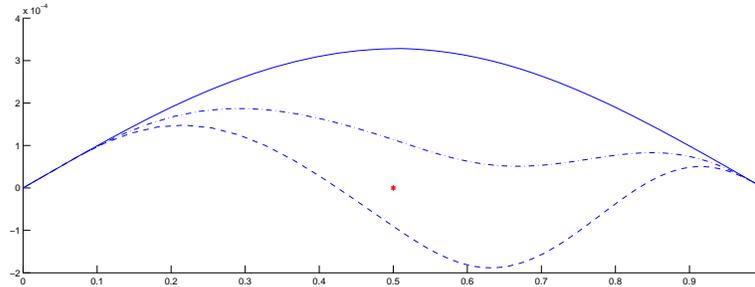
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Figure 3: Red star is the obstacle, solid line for $\gamma = 0$, the dash-dotted line for $\gamma = 0.5$ (middle one), and the dashed line is for $\gamma = 1$.

the problem a "keeping the right" cost functional, that is $\psi(X_0, X_f, X, t) = y_T(t)$ and we increase the weight γ given to this psychological factor we have that the pedestrian chose the right side of the obstacle (dashed line in Figure 3).

5   Conclusions

In this paper we addressed the modeling problem for the motion of a single pedestrian. In particular we focus on the need to design models taking into account both the psychological aspects and the mechanicistic aspects of human locomotion. We review the wide psychological literature on the cognitive processes involved in walking and the scientific literature (mostly coming from applied mathematics, physics, and engineering) on quantitative aspects. Then, we analyse the experimental settings for model testing and we propose two experiments in a natural environment. In the first experiment the results show the strong influence of the experimental setting, well beyond possible random fluctuations, on the pedestrians' behavior. The second one to determine the mechanicistic and the cognitive aspects involved in human locomotion. Finally, we propose a cognitive model combining psychological insight with optimization models from robotics.

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